Analysis of Vertical Pneumatic Conveying of Solids Using Multiphase Flow Models

The multiphase flow models were used to predict pressure drop and segregation of particles flowing in a vertical pipe. To compare calculations with data, it was necessary to assume an effective particle size based on a coefficient of restitution and on inlet void fractions. Different partifle size distributions with equal average particle size generate distinctly different pressure drops and segregations. Contribution of solid interaction force is very important in accounting for the segregation of the particles in a vertical pipe.

All models gave a reasonable prediction of the design parameters. The pressure drops predicted by the models agreed well with both high- and low-pressure experiments.

HAMID ARASTOOPOUR, SHAW-CHAN LIN and S. A. WEIL

Department of Gas Engineering
Illinois Institute of Technology/Institute of Gas Technology
Chicago, IL 60616

SCOPE

For the design of vertical solids pneumatic transport systems, the literature offers several empirical correlations and hydrodynamic models. The former are limited to the particular data base and the latter are generally limited to narrow particle size distributions. To design for the transport of solids of wide distribution requires a proper accounting of both the segregation of particles and its effects on pressure drop and choking limits. A hydrodynamic model of such systems must consider the solids as a component of many phases. If the solids are assumed to consist of a finite number of solid phases, previously developed two- or three-phase hydrodynamic models can be extended to the more complex systems. Furthermore, it is apparent that interaction between particles should be considered in the model. The present work is directed along these paths.

CONCLUSIONS AND SIGNIFICANCE

In this work, a multiphase hydrodynamic model was developed for vertical pneumatic transport of solids to account for the effects due to particle size distribution. The model includes terms to account for particle interaction. The potential of predictions of phenomena such as choking, solids segregation, and minimum pressure drop was shown, and the dangers of using a mean particle size in a system of wide distribution were demonstrated. The appropriateness of the model was shown by comparison of calculated and reported pressure drops in systems of widely different operating pressures. The studies indicate the importance of and therefore the need for better estimates of particle interaction forces and voidage to relate segregation and pressure drop to the operating parameters of flow rates, size distribution, and pressure.

INTRODUCTION

In the literature there are many correlations for the design parameters of vertical solids transport systems, including those of Zenz (1949), Leung (1976), and Knowlton and Bachovchin (1976). Most of the correlations are probably applicable only to particular conveying systems; accordingly, a lot of disagreement over their general validity is evident. To obtain the generalized formulation of the behavior of gas and solids in such systems, a more fundamental approach based on fluid mechanical and thermodynamic principles is essential. Hydrodynamic equations that are discussed by some investigators such as Soo (1967), Vernier and Delhaye (1968), Jackson (1971), and Ishii (1975) were applied by Arastoopour and Gidaspow (1979) for the two-phase-flow system of gas as one phase and of average uniform particles as the other phase for vertical pneumatic conveying of the particles. Their calculated values agreed well with measured values such as pressure drop for narrow particle size distributions. Those two-phase-flow models do not incorporate the effect of particle size distribution and interaction and cannot account for solids segregation, an important characteristic of gas-solids flow. For binary solid mixtures, three-phase hydrodynamic models have been employed by Nakamura and Capes (1976) and Arastoopour, Lin, and Gidaspow (1978). In addition to pressure drop and choking velocity, they also analyzed the segregation of the binary solid mixture. Yang (1978) also developed a segregation model for particles, based on the continuity equations of solids, particle velocity, and pressure drop due to solid friction. In this study, Arastoopour and Gidaspow's (1979) two-phase analysis of the gas-solid flow is extended to develop multiphase hydrodynamic equations. Different sizes of particles are considered as separate phases, and particle interaction forces are included.

SOLID-GAS MODELS

The vertical pneumatic conveying of particles of different size distributions is described by one-dimensional, isothermal, steady-state multiphase hydrodynamic equations. The following assumptions were made:

1. There is no interphase mass/heat transfer.
2. The particle wall friction was neglected due to dilute solid phases.
3. The total voidage was used for the drag force expressions between the gas and each of the solid particle phases.

As pointed out by Arastoopour and Gidaspow (1979), in the concentrated solids-gas vertical flow, the existence of clumps should be considered a result of trapping of particles of different size,

All correspondence concerning this paper should be addressed to H. Arastoopour.

ISSN-0001-1541-82-5665-0467-$2.00.

collision of the particles with the walls and with each other, and interaction between the boundary layers of the individual particles. Clumps may also occur if the particles are sticky. To account for the clumps, an average effective diameter for the particles was defined. In this study, the average effective diameter for individual particles was assumed as follows:

$$d_{ei} = \frac{d_i}{e}$$

The parameter e is related to the coefficient of restitution. As a first attempt it was assumed that e is identical to the coefficient of restitution. The continuity equations and the mixture momentum balance can be written as Ishii, 1975; Banerjee and Chan, 1980:

Gas Phase Continuity:
$$\frac{d}{dx}[\epsilon_g \rho_g V_g] = 0 \tag{1}$$

Solid Phase i Continuity:
$$\frac{d}{dx}[\epsilon_i \rho_i V_i] = 0 \qquad i = 1, 2, \ldots n \tag{2}$$

Mixture Momentum Equation:

$$\epsilon_g \rho_g V_g \frac{dV_g}{dX} + \sum_{i=1}^{n} \epsilon_i \rho_i V_i \frac{dV_i}{dX} + \frac{dP}{dX} = -\left[\epsilon_g \rho_g + \sum_{i=1}^{n} \epsilon_i \rho_i\right] g - f_w \tag{3}$$

The gas was assumed ideal, and the solids were incompressible.

In order to determine the gas velocity V_g, the solids velocities V_i, the system pressure P, and the volume fractions ϵ_g and ϵ_i, we need $2n + 2$ equations in addition to Eq. 4.

$$\epsilon_n = 1 - \epsilon_g - \sum_{i=1}^{n-1} \epsilon_i \tag{4}$$

From continuity equations and the mixture momentum equation, we have already had $n + 2$ equations. The n remaining equations are the momentum equations for the n solid phase, which have been proposed in different forms.

Solid Momentum Equations:

$$\rho_i V_i \frac{dV_i}{dx} + \eta \frac{dP}{dx} + \gamma \frac{P}{\epsilon_i}\frac{d\epsilon_i}{dx} = f_i - \rho_i g - \sum_{\substack{j=1 \\ j \neq i}}^{n} I_{ij\, i=1,2,\ldots n} \tag{5}$$

The distribution coefficient was assumed to be unity in Eqs. 1 through 5. The different values of the parameters, η and γ, correspond to different cases:

Case (A): $\eta = 1$, $\gamma = 0$. Pressure Drop in the Solids and Gas Phases Model.

Case (B): $\eta = 0$, $\gamma = 0$. Pressure Drop in Gas Phase Only Model.

Case (C): $\eta = 1$, $\gamma = 1$. Partial Pressure Drop in Solids and Gas Phases Model.

Parameter γ is similar to the displacement factor, B, presented in Soo's (1979) transient one-dimensional momentum equation for multiphase flow. The displacement factor is dependent upon the flow configuration, which affects the diffusion of a phase. Some special flow configurations classified by Soo are—

a. Flow with Brownian Motion: $B = 0$ or $\gamma = 1$
b. Dispersed Flow: $B \rightarrow 0$ or $\gamma \rightarrow 1$
c. Slug Flow: $1 > B > 0$ or $0 < \gamma < 1$
d. Wavey Stratified Flow: $B < 1$ or $\gamma > 0$
e. Purely Stratified Flow: $B = 1$ or $\gamma = 0$
f. Distributed Stratified Flow: $B = 1$ or $\gamma = 0$.

RELATIVE VELOCITY MODEL

Based on Gidaspow's (1976) one-dimensional relative velocity model, the multiphase one-dimensional relative velocity model with unit distribution coefficient can be extended as follows:

$$-\frac{1}{2}\frac{d}{dX}(V_g - V_i)^2 = \frac{f_i}{\rho_i} - g - \frac{1}{\rho_i}\sum_{\substack{j=1 \\ j \neq 1}}^{n} I_{ij\, i=1,2,\ldots n} \tag{6}$$

FRICTION FORCES

In the above equations, f_w and f_i are gas wall friction and drag force per unit volume of particle i, respectively. Similar to Arastoopour and Gidaspow (1979), the usual Fanning's equation for gas wall friction was used. The drag force, f_i, exerted by the gas on a unit volume of particle i can be written as (Richardson and Zaki, 1954):

$$f_i = \frac{3}{4} C_{Di} \frac{\rho_g (V_g - V_i)}{d_{ei}} |V_g - V_i| \, \epsilon_g^{-2.65} \tag{7}$$

The above correlation for the drag force is valid for spherical particles. The drag coefficient, C_{Di}, can be related to the Reynold's number (Rowe, 1961) by means of the following relations:

$$C_{Di} = \frac{24}{R_{ei}}(1 + 0.15\, R_{ei}^{0.687}) \qquad R_{ei} < 1000 \tag{8}$$

$$C_{Di} = 0.44 \qquad R_{ei} > 1000 \tag{9}$$

where

$$R_{ei} = \epsilon_g \rho_g d_{ei} |V_g - V_i| / \mu_g \tag{10}$$

PARTICLE-PARTICLE INTERACTION

The particle-particle interaction, I_{ij}, which results from the collisions between particles with different velocities, was derived from the conservation of linear momentum and the consideration that relative velocity after collision is equal to the relative velocity before collision times the coefficient of restitution (Nakamura and Capes, 1976; Lin, 1980). For the multisize particle mixtures, the momentum transfer between particle i and particle j per unit volume of particle i can be written as:

$$I_{ij} = \frac{3}{4}\alpha(1 + e)\frac{\rho_i \rho_j \epsilon_j (R_i + R_j)^2}{\rho_i R_i^3 + \rho_j R_j^3}(V_i - V_j)^2 \tag{11}$$

where e is the coefficient of restitution. The collisions between particles are plastic when $e = 0$, and the collisions are elastic when $e = 1$. The value of α accounts for non-head-on collision and multiple scattering, and it could be a function of particle size and particle density.

Equations 1 through 4 with 5 or 6 are sets of $(2n + 3)$ nonlinear first-order differential equations. The Runge-Kutta method was used to obtain the numerical results for the above sets of differential equations as an initial value problem.

PARAMETRIC STUDY USING RELATIVE VELOCITY MODEL

To use actual numerical values and be able to compare the calculated results from the proposed models with the experimental data, the vertical pneumatic transport studied here is the same as that used by Knowlton and Bachovchin (1976) in their study of vertical pneumatic conveying of a wide distribution of particles which is shown in Table 1. The diameter of the conveying pipe was 7.36×10^{-2} m, and its length was 15.2 m. The experiments were run with nitrogen and a lignite char of density 1.26×10^3 kg/m^3. In Knowlton and Bachovchin's experiment, the superficial gas velocity U_g, the solid mass flow rate W_s, inlet pressure, and the pressure drop at different sections of the pipe were measured.

To compare the experiment with the calculated values, the multiphase flow models also need an initial value for the void fraction and values for α and e. Because these were not reported, reasonable values are selected for each parameter. The calculated values are not sensitive to values of α; therefore $\alpha = 1$ was assigned

TABLE 1. PARTICLE SIZE DISTRIBUTION OF LIGNITE CHAR

Particles Diameters (m × 10^5)	Weight, %
6.71	9.2
16.95	13.4
24.87	21.5
41.94	15.6
59.44	13.5
84.86	20.6
144.78	6.2

for all calculations. The particles of different size ranges were designated *A, B, C, D, E, F*, and *H* according to their increasing diameters.

Figure 1 shows the particle velocities calculated using the relative velocity model at $W_s = 100.38$ kg/m$^2\cdot$s, $U_g = 15.24$ m/s, $P_1 = 4.824 \times 10^5$ N/m^2, $\epsilon_{gl} = 0.9$, $e = 0.1$, and $\alpha = 1.0$.

The larger particles are accelerated not only by the gas but also by small particles due to momentum transfer caused by collisions between particles. The effect of different values of e was calculated at the initial pressure of 4.824×10^5 N/m^2, the initial gas volume fraction of 0.9, and the inlet superficial gas velocity of 15.24 m/s, and the values of α was chosen to be 1. As physically expected, the smaller values of e result in larger effective particle diameters and lower solids velocities and thus higher values for pressure drop. At a lower value of e, the acceleration zone is longer, and the particle velocities are lower due to the larger effective particle sizes. The lower particle velocities result in a higher particle volume fraction at constant mass flow rate, which in turn causes additional pressure drop in the vertical line.

At any values of initial superficial gas velocities, the smaller particles have higher velocities than those of larger particles due to the more effective drag forces. As initial superficial gas velocity increases, the particles continue to accelerate because of higher relative velocities between the gas and solid phases; therefore, the solid velocities are higher than those at lower gas velocities. Following Nakamura and Capes (1976), the segregation along the pipe can be expressed by the solid volume fraction ratio, x_i/X_i:

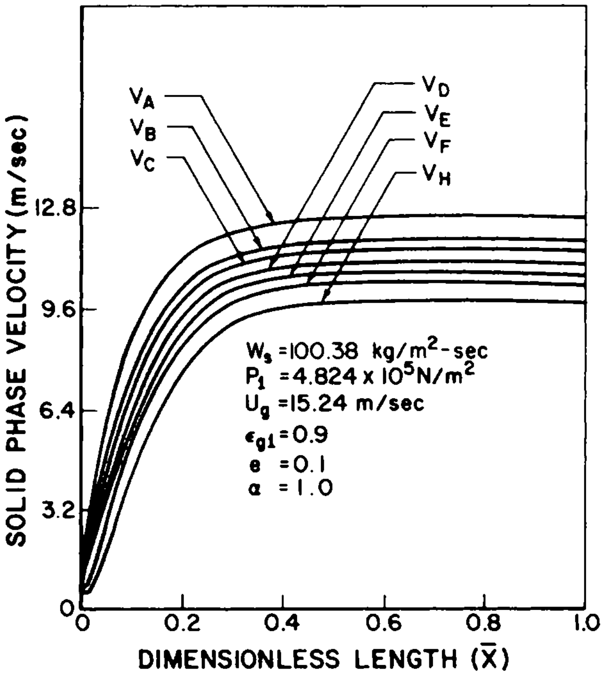

Figure 1. Solid-phase velocities for solids-gas flow through vertical pipe calculated using relative velocity model with particle interaction consideration.

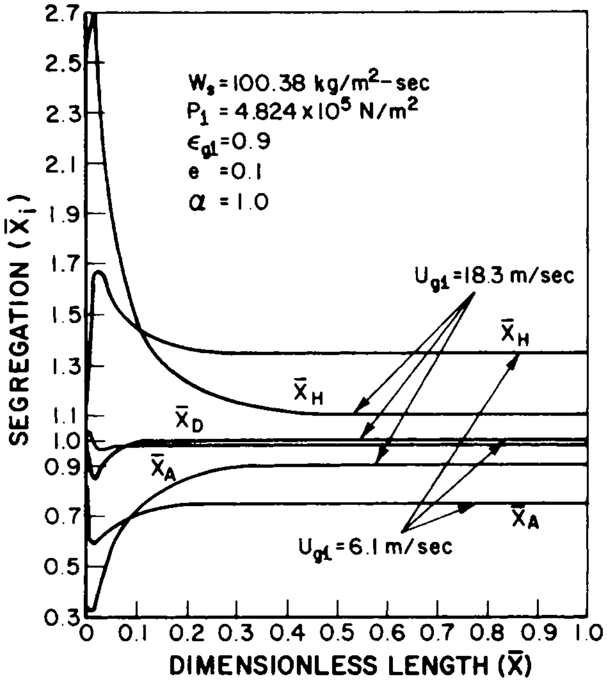

Figure 2. Effect of initial superficial gas velocity on solid segregation calculated using relative velocity model.

$$x_i/X_i = (1 - \epsilon_{gl})\left(\epsilon_{il} + \sum_{\substack{j=1\\ j\neq i}}^{n-1} \epsilon_{jl}\frac{\bar{V}_i}{\bar{V}_j}\right) \tag{12}$$

The above expression indicates that the segregation is determined by the combination of solid velocity ratios. At lower superficial gas velocities, the smaller particles have significantly greater accelerations than the larger particles. At higher initial gas velocities, the drag force exerted on small and large particles is sufficient to accelerate all the particles in a short zone and results in high velocities for all the particles and hence a velocity ratio of the order of unity. Therefore, the segregation of particles is more complete at lower initial gas velocities than that at higher initial gas velocities. Figure 2 shows this behavior clearly.

Figure 3 shows the effect of initial pressure on the segregation

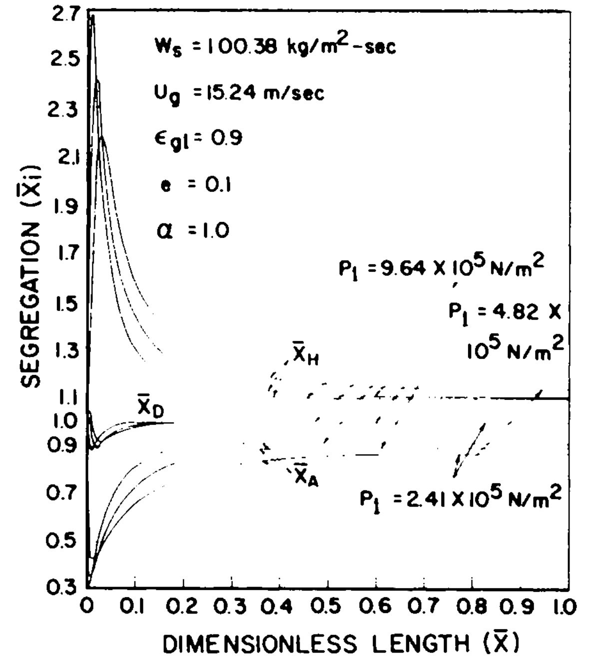

Figure 3. Effect of initial pressure on solid segregation calculated using relative velocity model.

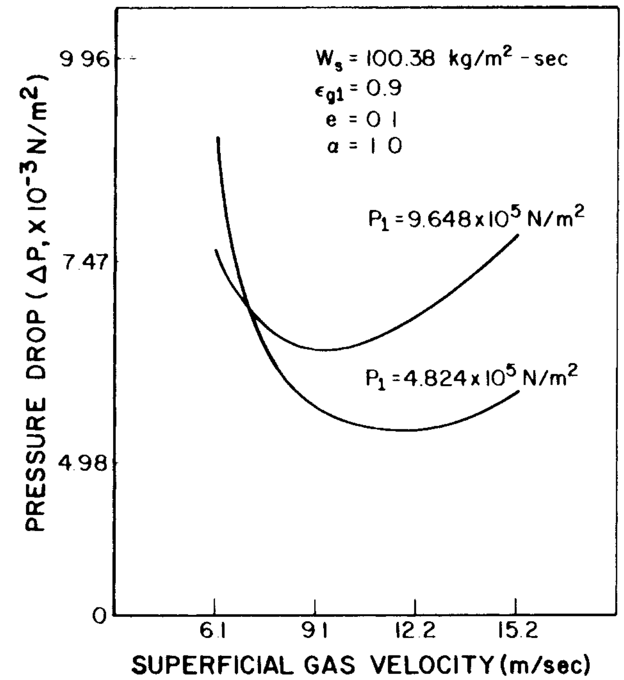

Figure 4. Effect of initial pressure on pressure drop at different superficial gas velocities calculated using relative velocity model.

of solid mixtures through the conveying line. The maximum segregation occurs at the beginning of the acceleration zone where the particle interaction is not significant compared to the other forces and where the smaller particles are accelerated more due to the larger drag force. After the acceleration zone, the segregation of each size of particle approaches the constant value. Lower inlet pressure (that is, lower gas density) yields lower drag forces. At low inlet pressure, the velocity ratios are far from unity, giving more complete segregation than at higher inlet pressure.

Figure 4 shows the pressure drop across the conveying line at different superficial gas velocities with different inlet pressures. The minimum in the pressure drop is the result of the balance between the wall frictional force and the solid static head. The increase in initial pressure generates a higher value for minimum pressure drop at lower gas velocities. At sufficiently low gas velocities, the frictional and velocity heads are small compared with the solid static head. A lower system pressure results in a lower gas density and thus decreases the drag force exerted on the solids by the gas. The decrease in drag force results in lower particle velocities and a longer acceleration zone, which in turn results in a higher solid volume fraction throughout the line and generates a higher pressure drop.

At higher gas velocities, the dominating force for pressure drop is gas wall friction. The higher initial pressure results in higher gas density and thus higher gas gravity term and gas momentum, which in turn generates higher pressure drop. This behavior is clearly shown in Figure 4. The effect of initial void fractions on the pressure drop across the conveying line is shown in Figure 5. At low void fraction, the particles move with lower velocities, which result in a longer acceleration zone and greater solid phase concentration accounting for the higher pressure drop.

The particle segregations at different values of initial gas volume fractions show a maximum segregation at the pipe entrance due to different drag forces exerted by the gas and less pronounced particle collisions. When the difference between particle velocities increases, the particle interaction forces become important; faster particles are slowed down, and slower particles are accelerated. After the acceleration zone, all particles reach constant velocities, and segregation remains constant regardless of different initial gas volume fraction. Therefore, the initial gas volume fraction does not have a significant effect on solids segregation outside the acceleration zone. For a more detailed parametric study, see Lin (1980).

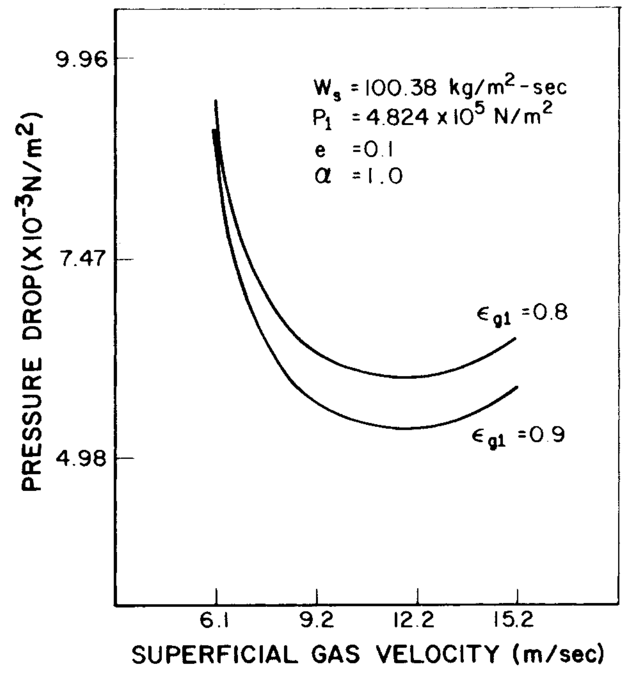

Figure 5. Effect of initial gas volume fraction on pressure drop at different superficial gas velocities calculated using relative velocity model.

EFFECT OF PARTICLE SIZE DISTRIBUTION

To study the effect of particle size distribution in pneumatic conveying of solids, three different, wide-range, size distributions with the same average size and particle density of 1.26×10^3 kg/m^2 were chosen. Table 2 shows these particle distributions with their corresponding volume fractions.

The pressure drop along the conveying line for different particle distributions was calculated using the relative velocity model with initial gas volume fraction of 0.9, inlet pressure of 4.824×10^6 N/m^2, and constant values for e and α of 0.1 and 1.0, respectively. Figure 6 shows the pressure drop variation at different superficial gas velocities for different particle distributions.

When the particle distributions are not too different, the pressure drop variation with superficial gas velocities is similar. This behavior is shown clearly in Figure 6 for particle size distribution of Systems I and II. However, when the percentage of the larger particles is higher, as in the case of System III, the pressure drop is higher at all superficial gas velocities. The minimum pressure drop is also at a higher superficial gas velocity. The difference in pressure drop is even more significant near the choking region.

TABLE 2. DIFFERENT PARTICLE DISTRIBUTIONS WITH SAME AVERAGE PARTICLE SIZE

Particle Diameter (m × 10^5)			Weight, %		
I	II	III	I	II	III
6.71	6.06	4.32	9.2	7.0	5.2
16.95	16.98	14.99	13.4	21.0	6.3
24.87	24.90	41.94	21.5	20.0	20.0
41.94	41.94	59.44	15.6	10.0	10.0
59.44	59.44	84.08	13.5	10.0	20.0
84.86	84.08	140.97	20.6	18.0	5.0
144.78	153.60	199.90	6.2	14.0	23.5
36.3*	36.0*	36.6*			

* Average Size: $d_p = \dfrac{1}{\sum \dfrac{X_i}{d_i}}$

X_i = volume fraction

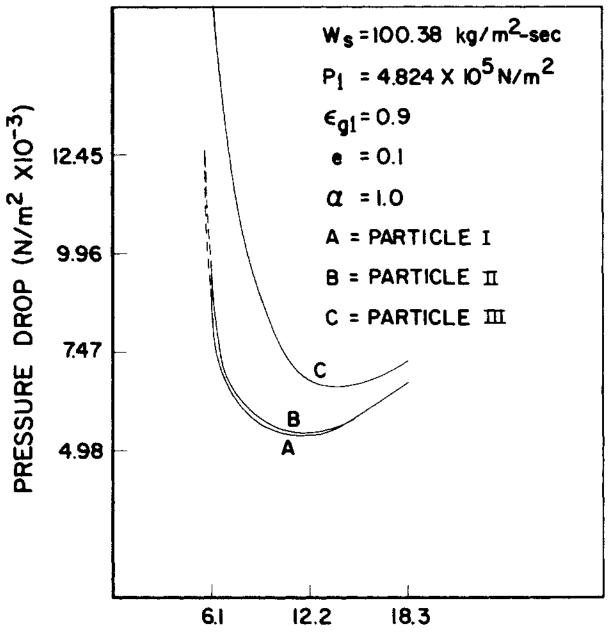

Figure 6. Effect of different particle size distribution with same average particle size on pressure drop at different superficial gas velocities calculated using relative velocity model.

Hence, it can be seen clearly that a great variation in pressure drop and choking velocity due to variation in particle distributions may be overlooked in correlations based upon average particle size. Therefore, choking and the minimum pressure drop points, both being important to the design of conveying lines, may occur at a higher superficial gas velocity than those estimated with the available correlations or hydrodynamic models based on the average particle size.

COMPARISON OF UNEQUAL VELOCITY MODELS

The calculated pressure drop and phase velocities based on different models for flow of nitrogen and lignite char with wide particle sizes through a vertical conveying line were compared at the conditions of the Knowlton and Bachovchin experiment (1976). The solid mass flow rate is 100.38 kg/m² · s, and the initial pressure is 4.824×10^5 N/m². The initial superficial gas velocity is 15.24 m/s with an assumed value of initial gas volume fraction equal to 0.9 and constant values for e and α of 0.1 and 1.0 respectively. The three different models considered are: (A) the relative velocity; (B) the pressure drop in gas phase only; and (C) the pressure drop in each phase. All three models show the experimentally observed choking behavior and a minimum in pressure drop. However, the relative velocity model predicts a higher pressure drop at all superficial gas velocities than Cases B and C, which indicates that more interaction forces between the gas and solid phases have been taken into account in the relative velocity model. Also, the relative velocity model predicts lower velocity for small particles and higher velocity for larger particles as compared with Cases B and C. The partial pressure drop in the solid and gas phase model (Case C) did not generate experimentally observed flow behaviors for gas and solids.

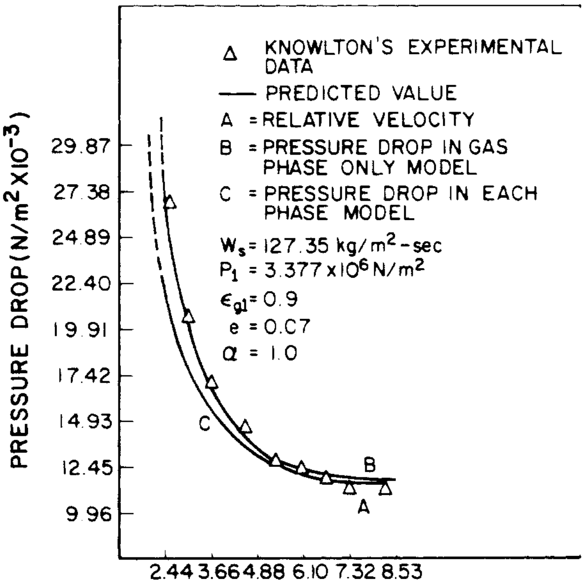

Figure 7. Comparison of Knowlton-Bachovchin's (1976) experimental results for lignite char with numerical solutions calculated using three unique velocity models.

COMPARISON WITH PRESSURE DROP EXPERIMENTAL DATA

To compare our calculated pressure drop with different experiments at either high or low pressures, the experimental values of the initial conditions of gas and solid particles in addition to the dimensions of the experimental setup were used. For high-pressure systems, the only available data is Knowlton and Bachovchin's experiment (1976) with a wide distribution of particle size. For low-pressure systems in the literature, there have been more data on pressure drop for pneumatic conveying of the solids such as Vogt and White (1948), Zenz (1949), Hariu and Malstad (1949), Belden and Kassel (1949), Metha, Smith and Comings (1957), Jones et al. (1967), Konno and Saito (1969), and Konchesky et al. (1975). Except for Zenz (1949) and Vogt and White, (1948) the other investigators did not report the particle distribution, which is one of the important factors in flow behavior of the pneumatic conveying of solids and is also a necessary and essential variable to our model. Thus, Zenz's experiment was chosen for comparison with predicted pressure drop.

COMPARISON WITH KNOWLTON AND BACHOVCHIN'S HIGH-PRESSURE EXPERIMENT

Knowlton and Bachovchin (1976) studied the effect of high pressure on the pressure drop and choking velocity in a nitrogen-lignite char, vertical pneumatic conveying line. The size distribution is shown in Table 1. The length of the conveying line was 15.24 m; the diameter of the pipe was 7.36×10^{-2} m. The inlet pressure, superficial gas velocities, and pressure drop over different sections of the pipe were reported. To generate the calculated results from the multiphase models, the inlet gas volume fraction was assigned as 0.9. The value of e as 0.07 for lignite was determined by comparing the predicted pressure drop with experiment at U_g = 6.1 m/s. The value of α was assigned as 1 due to insensitivity of pressure drop variation with α.

TABLE 3. PARTICLE CHARACTERISTICS IN ZENZ'S EXPERIMENT

Particle Diameter (m × 10⁵)	Weight, %
7.364	2.2
8.888	13.6
12.448	11.0
14.984	27.0
17.782	5.8
24.890	40.2
41.910	0.2

Particle Density: kg/m³ = 2.098×10^3

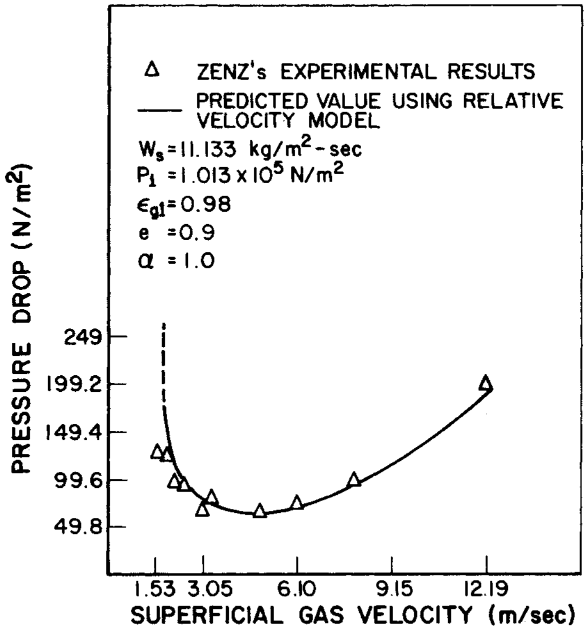

Figure 8. Comparison of pressure drop calculated using multiphase relative velocity model with Zenz's (1949) experimental results.

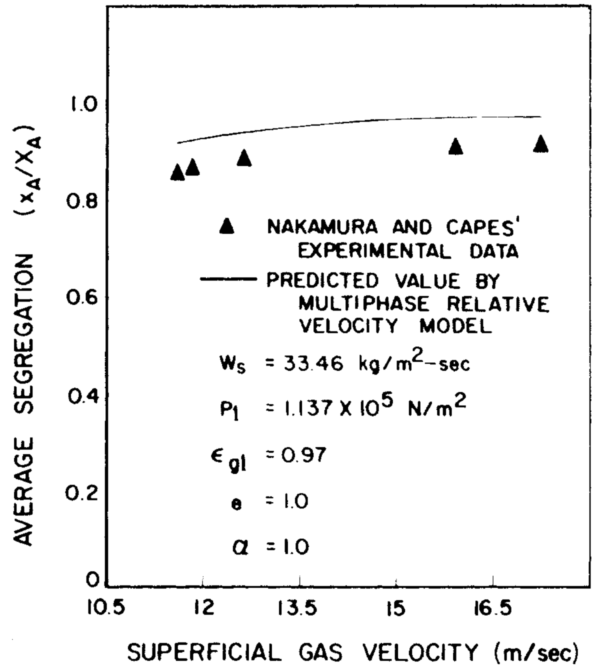

Figure 9. Comparison of average segregation calculated using multiphase relative velocity model with Nakamura and Capes' (1976) experimental results.

Figure 7 shows the calculated total pressure drop using the three models: (A) relative velocity; (B) pressure drop in gas phase only; and (C) pressure drop in solid and gas phases for nitrogen and lignite char vertical flow. Figure 7 shows that Cases (A) and (B) show better agreement for pressure drop with the experimental data. Case (C) compares well with pressure drop experimental values at high gas superficial velocities, but it deviates from the experiment at low gas superficial velocities. Generally speaking, all three models describe reasonably well the Knowlton and Bachovchin experimental data (1976) as shown in Figure 7.

COMPARISON WITH ZENZ'S LOW-PRESSURE EXPERIMENT

In Zenz's experiment, air was used as gas phase, and salt was used as solid phase. Table 3 shows the particle characteristics. The diameter of the pipe was 4.45×10^{-2} m; the length of the test section was 1.118 m. Zenz did not report the inlet pressure and inlet gas volume fraction. An appropriate initial low-pressure value of 1.03×10^5 N/m^2 was chosen, and the initial gas volume fraction was assigned to be 0.98. The predicted pressure drop calculated using the relative velocity model fitted the experimental result for $W_s = 11.133$ kg m$^2 \cdot$ s at $U_g = 7.65$ m/s with $e = 0.9$, $\alpha = 1.0$, and the pipe relative roughness of 0.05. Figure 8 shows the calculated values using the relative velocity model. It can be seen in Figure 8 that the calculated pressure drop using the multiphase relative velocity model compared well with the experimental results at all superficial gas velocities.

COMPARISON WITH NAKAMURA AND CAPES' SEGREGATION DATA

In Nakamura and Capes' experiment, the gas phase was air and the solid phases were spherical glass beads of two different sizes. One solid phase has the density of 2.90×10^3 kg/m^3 and particle diameter of 1.08×10^{-3} m. The other one has a density of 2.86×10^3 kg/m^3 and the particle diameter of 2.90×10^{-3} m. The diameter of the conveying line was 1.18×10^{-2} m and its length was 9.144 m.

From the experiment, the solid mass flow rate $W_s = 33.46$ kg/m$^2 \cdot$ s, the superficial gas velocity U_g, the average segregation in the test section x_A/X_A, the solid volume fraction in the feed $X_A = 0.767$, and the pressure drop along the test section were measured. In our numerical calculation, a gas volume fraction and the pressure at inlet or any arbitrary point are needed. Since such data were not given in the cited experiment, values for initial pressure and inlet gas volume fraction were assumed. The effective particle diameter was considered to be identical to the actual diameter of particles. The experimental values for the pressure drop agree well with the calculated values at a gas void fraction of $\epsilon_{g_1} = 0.97$ and initial pressure of $P_1 = 1.137 \times 10^5$ N/m^2. Using these values for ϵ_{g_1} and P_1, the calculated values using the relative velocity model showed a good agreement with Nakamura and Capes' segregation data. Figure 9 shows a comparison of particle segregation with their experiment using the relative velocity model. The slight deviation could be due to improper values for ϵ_g, neglect of wall friction effect, and the validity of assumption in binary collisional friction forces in our model. An independent measurement of such particle-particle interaction forces is needed to refine the model.

NOTATION

B	= displacement factor
C_{Di}	= drag coefficient of solid phase i
d_i	= diameter of particle i
d_{ei}	= effective diameter of particle i
e	= coefficient of restitution
f_i	= drag force per unit volume of solid phase i
f_w	= gas wall friction force
g	= gravity acceleration
I_{ij}	= momentum transfer per unit volume of particle i between particles i and j
i	= solid phase i
j	= solid phase j
n	= number of solid phases
P	= pressure
P_i	= initial pressure
R_{ei}	= Reynolds number of solid phase i
U_g	= superficial gas velocity
V_i	= solid phase i velocity
$\overline{V}_i$	= dimensionless solid phase i velocity, V_i/V_{i_1}
V_g	= gas velocity
W_s	= solid mass flow rate
X	= coordinate parallel to flow direction

$\overline{X}$ = dimensionless coordinate, X/L
x_i = solid volume concentration of phase i
$\underline{X}_i$ = solid volume concentration of solid phase i in the feed
$\overline{X}_i$ = dimensionless solid phase i segregation, x_i/X_i

Greek Symbols

α = constant which accounts for non-head-on collision and multiple scattering
η = parameter in solid-phase momentum equations
γ = parameter in solid-phase momentum equations
ϵ_i = volume fraction of solid phase i
ϵ_n = volume fraction of the nth solid phase
ϵ_g = gas volume fraction
ϵ_{g1} = initial gas volume fraction
μ_g = gas viscosity
ρ_i = density of solid phase i
ρ_g = density of gas

LITERATURE CITED

Arastoopour, H., and D. Gidaspow, "Vertical Pneumatic Conveying Using Four Hydrodynamic Models," *Ind. and Eng. Chem.*, **18**, 123 (1979).

Arastoopour, H., and D. Gidaspow, "Analysis of IGT Pneumatic Conveying Data and Fast Fluidization Using a Thermodynamic Model," *J. of Powder Tech.*, **22**, No. 1, 77 (1979).

Arastoopour, H., and D. Gidaspow, "Two Phase Flow Fundamentals," in *Two Phase Transport and Reactor Safety 1*, 133, Hemisphere Publishing Corp., Washington (1978).

Arastoopour, H., D. Lin, and D. Gidaspow, "Hydrodynamic Analysis of Pneumatic Transport of a Mixture of Two Particle Sizes," in *Multiphase Transport*, 1853, Hemisphere Publishing Corp., Washington (1978).

Banerjee, S. and A. M. C. Chan, "Separated Flow Models-I, Analysis of the Averaged and Local Instantaneous Formulations," *Int.J. Multiphase Flow*, **6**, 1 (1980).

Belden, D. H., and L. Kassel, "Pressure Drops Encountered in Conveying Particles of Large Diameter in Vertical Transfer Line," *Ind. and Eng. Chem.*, **41**, 1174 (1949).

Gidaspow, D., and C. W. Solbrig, "Transient Two-Phase Flow Models in Energy Production," State-of-the-Art Paper presented at the AIChE 81st National Meeting (April 11–13, 1976); in revised form preprinted for NATO Advanced Study Institute on Two-Phase Flows and Heat Transfer, ASI Proceedings, Istanbul, Turkey (August 16–27, 1976).

Hariu, O. H., and M. C. Molstad, "Pressure Drop in Vertical Tubes in Transport of Solids by Gases," *Indust. and Eng. Chemistry*, **41**, 1148 (1949).

Ishii, M., *Thermo-Fluid Dynamic Theory of Two Phase Flow*, Eyrolles, Paris (1975).

Jackson, R., "Chapter 3, Fluid Mechanical Theory," *Fluidization*, 65, J. R. Davidson, and D. Harrison, eds., Academic Press, New York (1971).

Jones, J. H., W. G. Braun, T. E. Daubuert, and H. D. Allendorf, "Estimation of Pressure Drop for Vertical Pneumatic Transport of Solids," *AIChE J.*, **13**, 608 (1967).

Knowlton, T. M., and D. M. Bachovchin, "The Determination of Gas-Solid Pressure Drop and Choking Velocity as a Function of Gas Density in a Vertical Pneumatic Conveying Line," *Fluidization Technology*, **2**, 253, D. Keairns, ed., Hemisphere Publishing Corp., Washington (1976).

Konchesky, J. L., T. J. George, and J. C. Craig, "Air and Power Requirement for the Pneumatic Transport of Crushed Coal in Vertical Pipelines," *Trans. of ASME*, **97**, 101 (1975).

Konno, H., and S. Saito, "Pneumatic Conveying of Solids Through Straight Pipes," *J. of Chem. Eng. of Japan*, **2**, 211 (1969).

Leung, L. S., and R. J. Wiles, "A Quantitative Design Procedure for Vertical Pneumatic Conveying Systems," *Ind. Eng. Chem. Process Des. Dev.*, **15**, No. 4 (1976).

Lin, S. C., "Hydrodynamic Analysis of Vertical Solids Transport Using Multiphase Flow Models," M.S. Thesis, Illinois Institute of Technology, Chicago (1980).

Mehta, N. C., J. M. Smith, and E. W. Comings, "Pressure Drop in Air-Solid Flow Systems," *Indust. Eng. Chemistry*, **49**, 986 (1957).

Nakamura, K., and C. E. Capes, "Vertical Pneumatic Conveying of Binary Particle Mixtures," *Fluidization Technology*, **2**, 149, D. Keairns, ed., Hemisphere Publishing Corp., Washington (1976).

Richardson, J. F., and W. N. Zaki, "Sedimentation and Fluidization: Part I," *Trans. Instn. Chem. Engrs.*, **32**, 35 (1954).

Rowe, P. N., and G. A. Henwood, "Drag Force in a Hydraulic Model of a Fluidized Bed," *Trans. Institute of Chem. Eng.*, **39**, 43 (1961).

Soo, S. L., *Fluid Dynamics of Multiphase Systems*, Blaisdell Publishing Co., Waltham, MA (1967).

Soo, S. L., "On the Nature of Equations of Multiphase-Multidomain Mechanics," Paper for Presentation at the Second Multiphase Flow and Heat Transfer Symposium-Workshop, Miami (April 16, 1979).

Vernier, P., and J. M. Delhaye, "General Two Phase Theory Flow Equations Applied to the Thermodynamics of Boiling Nuclear Reactors," *Energie Primaire*, 4 (1968).

Vogt, E. G., and R. R. White, "Friction in the Flow of Suspension, Granular Solids in Gases Through Pipe," *Ind. Eng. Chem.*, **40**, 1731 (1948).

Yang, W. C., "Study on Segregation Modelling of Particles in Dilute Phase Vertical Pneumatic Transport Lines," The Fourth International Conference on the Pneumatic Transport of Solids in Pipes (June 26–28, 1978).

Zenz, F. A., "Two-Phase Fluid Fluid-Solid Flow," *Ind. and Eng. Chem.*, **41**, 2801 (1949).

Manuscript received September 2, 1980; revision received July 16, and accepted August 7, 1981.